2) Basis : $n = 1$

$f_n + f_{n+2} = l_{n+1} \rightarrow f_1 + f_3 = 1 + 2 = l_1 + l_0 = l_2$

$K \geq n$

Inductive Hypothesis:

$f_K + f_{K+2} = l_{K+1}$

Inductive Step:

① From the definition of Fibonacci number sequence we get

$f_{n+1} + f_{n+3} = f_n + f_{n-1} + f_{n+2} + f_{n+1}$

② Combining the two following equalities

$f_n + f_{n+2} = l_{n+1}$ and $f_{n-1} + f_{n+1} = l_n$, we get

$f_n + f_{n+2} + f_{n-1} + f_{n+1} = l_{n+1} + l_n$

③ From definition of Lucas number sequence, we get

$l_{n+1} + l_n = l_{n+2}$

3) a) R is

- reflexive
- symmetric
- not anti-symm
- transitive

∴ Is a equivalence relation

Is not a partial order

b) S is

- reflexive
- symmetric
- not anti-sym
- not transitive

∴ Is not an equivalence rel.

Is not a partial order

4) Must prove to be ① Reflexive, ② symmetric ③ Transitive

① $X - X = 0$; $0 \in Q$

② Assume $X - Y \in Q$

$Y - X = -(X - Y) = (-X + Y) =$ rational num.

③ $X - Y \in Q$

$Y - Z \in Q$

Adding them gives $X - Z$ which is also $\in Q$

and a rational num.